ROBUSTNESS ANALYSIS OF FLEXIBLE AIRPLANES

An advanced μ-analysis approach

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Introduction

For economical reasons, the size of civilian airplanes has recently increased quite significantly while limiting mass augmentation as far as possible.

Consequently, aircrafts have become "more flexible" so that :

- Flexible modes have progressively appeared in the control bandwidth,
- Active control has then become necessary to manage the interactions between these new modes and the "classical" ones which we associate to the rigid part of the aircraft.

Introduction

Such Active Control design methods are generally based on heuristic schemes. Moreover, the design is necessarily performed on a simplified model which only includes a few flexible modes.

Consequently, preservation of stability and performances - *i.e* **robustness** - on the full model has to be checked a posteriori.

The main objective of this talk is to present a new method to perform this Robustness Analysis which :

- avoid intensive Monte-Carlo simulations,
- provides guaranteed robustness margins,
- is not too conservative,
- still works on high-order systems with numerous uncertainties,
- runs as fast as possible for possible inclusion in a design process

Introduction

More precisely, the practical problem to be solved consists in analysing the robustness properties of a flight control system of a flexible transport aircraft with parametric uncertainties :

- in the rigid part of the model (aerodynamic uncertainties),
- in the natural frequencies of the bending modes

We want to compute an estimate of the maximal size of uncertainty which preserves stability. This is the Robustness Margin.

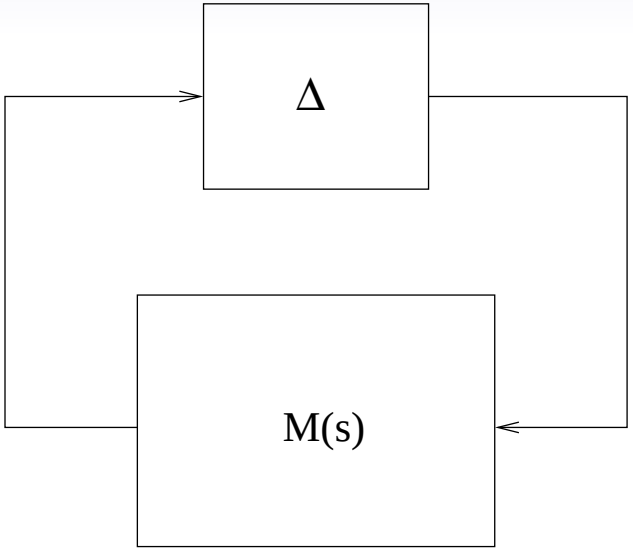
Our approach is based on the important notion of **Structured Singular Value** (SSV), denoted μ.

Outline

- Introduction to μ analysis
  - Standard interconnection structure
  - μ and the robustness margin
  - Main difficulties
  - Classical solutions versus ours
- Computation of a reliable μ upper-bound
  - Computing μ at a single frequency point
  - Computing μ at a several frequency points
  - Validity on a frequency segment
  - A proposed algorithm
- Application to the flexible aircraft
  - Model description and LFT format
  - Application of the routine
  - Results and comments
- Conclusion

Introduction to μ analysis

Standard interconnection structure



$$\Delta = \mathbf{diag}\,(\Delta_1(s), \ldots, \Delta_q(s), \delta_1 I_{n_1}, \ldots, \delta_r I_{n_r}) \in k\mathcal{B}_\Delta$$

where $\mathcal{B}_\Delta$ denotes the unit ball :

$$\mathcal{B}_\Delta = \{\Delta \,/\, \overline{\sigma}(\Delta) \leq 1\}$$

Introduction to μ analysis

Definition of μ

$$\begin{aligned}\mu_\Delta(M) &= 1/min(k \ / \ \exists \Delta \in kB\Delta \ with \ det(I - M\Delta) = 0) \\ &= 0 \ if \ no \ (k, \Delta) \ exists\end{aligned}$$

Robustness margin

$$\frac{1}{k_{max}} = \max_{\omega \in [0,\infty]} \mu(M(j\omega))$$

Introduction to μ analysis

Main difficulties

- The exact computation of μ is generally NP-hard,
- Evaluating the robustness margin involves a computation of μ for each frequency point inside the range of interest.

Classical solutions

- μ upper-bound computation via polynomial-time algorithms,
- Maximum value research over the frequency range :
  - by using a frequency gridding (not reliable in case of highly flexible modes),
  - by considering frequency as a repeated parametric uncertainty (not applicable for high order systems),
- Computation of a μ lower-bound to evaluate conservatism :
  - global μ lower-bound,
  - frequency-dependent μ lower-bound

Introduction to μ analysis

Our proposed solution : a general view

- Compute a "local" μ upper-bound using light adaptations of standard algorithms,
- Compute (by an **exact** method) all frequency segments where the above upper-bound is valid,
- Eliminate these frequency segments and go back to first step.
- Stop when the whole frequency range has been covered.
- Estimate conservatism by a global μ lower-bound computation

Computation of a reliable μ upper-bound

Computation at a single frequency point

LMI-based formulation :

Given scaling matrices $D \in \mathcal{D}$ and $G \in \mathcal{G}$:

$$M^* D M + j(GM - M^* G) \leq \beta^2 D \;\Rightarrow\; \mu_\Delta(M) \leq \beta$$

Singular-value based formulation :

Given scaling matrices $D \in \hat{\mathcal{D}}$ and $G \in \hat{\mathcal{G}}$, denote $F = I + G^2$, then :

$$\overline{\sigma}\left(F^{-1/4}\left(\frac{DMD^{-1}}{\beta} - jG\right)F^{-1/4}\right) \leq 1 \Rightarrow\; \mu_\Delta(M) \leq \beta$$

Computation of a reliable μ upper-bound

Remarks about the above two formulations

- Both formulations are equivalent.
- The first one leads to a convex optimization problem (gevp), and is implemented in LMI control toolbox of Matlab. This implementation is slow in case of largely repeated uncertainties,
- The second formulation forms the basis of an algorithm based on "power-iterations" and is implemented in the μ Analysis and Synthesis Toolbox. According to the options, this implementation may be more conservative. Yet, the computation is performed much faster,
- Both implementations are optimal for a fixed frequency point. Therefore, the scaling matrices D and G may not be robust versus frequency variations.

Computation of a reliable μ upper-bound

Simultaneous Computation at several frequency points

Consider two close frequency points ω_1 and ω_2, and denote M_1 and M_2 the associated frequency responses of the system.

Using the LMI-based formulation :

This formulation can be easily generalized. We now simply have two matrix inequalities instead of one :

$$\left\{ \begin{array}{l} M_1^* D M_1 + j(G M_1 - M_1^* G) \leq \beta^2 D \\ M_2^* D M_2 + j(G M_2 - M_2^* G) \leq \beta^2 D \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \mu_\Delta(M_1) \leq \beta \\ \mu_\Delta(M_2) \leq \beta \end{array} \right.$$

Computation of a reliable μ upper-bound

Simultaneous Computation at several frequency points

Using the SV-based formulation :

We propose a simple two-steps procedure :

- **Step 1 :** Using the power-algorithm, compute β_0, D and G on $M_0 = (M_1 + M_2)/2$,
- **Step 2 :** For fixed D and G, compute $\beta \geq \beta_0$ such that :

$$\overline{\sigma}\left(F^{-1/4}\left(\frac{DM_iD^{-1}}{\beta} - jG\right)F^{-1/4}\right) \leq 1 \quad (*)$$

by using a simple technical result (see next slide).

Computation of a reliable μ upper-bound

Simultaneous Computation at several frequency points

A technical result for Step 2

Denote $U_i = F^{-1/4} D M_i D^{-1} F^{-1/4}$ and $V = -jG(I + G^2)^{-1/2}$.
Further define $\hat{\beta}$ as the largest positive real such that :

$$det\left(\begin{bmatrix} 0 & U_i \\ U_i^* & 0 \end{bmatrix} + \hat{\beta}\begin{bmatrix} I & V \\ V^* & I \end{bmatrix}\right) = 0 \quad (**)$$

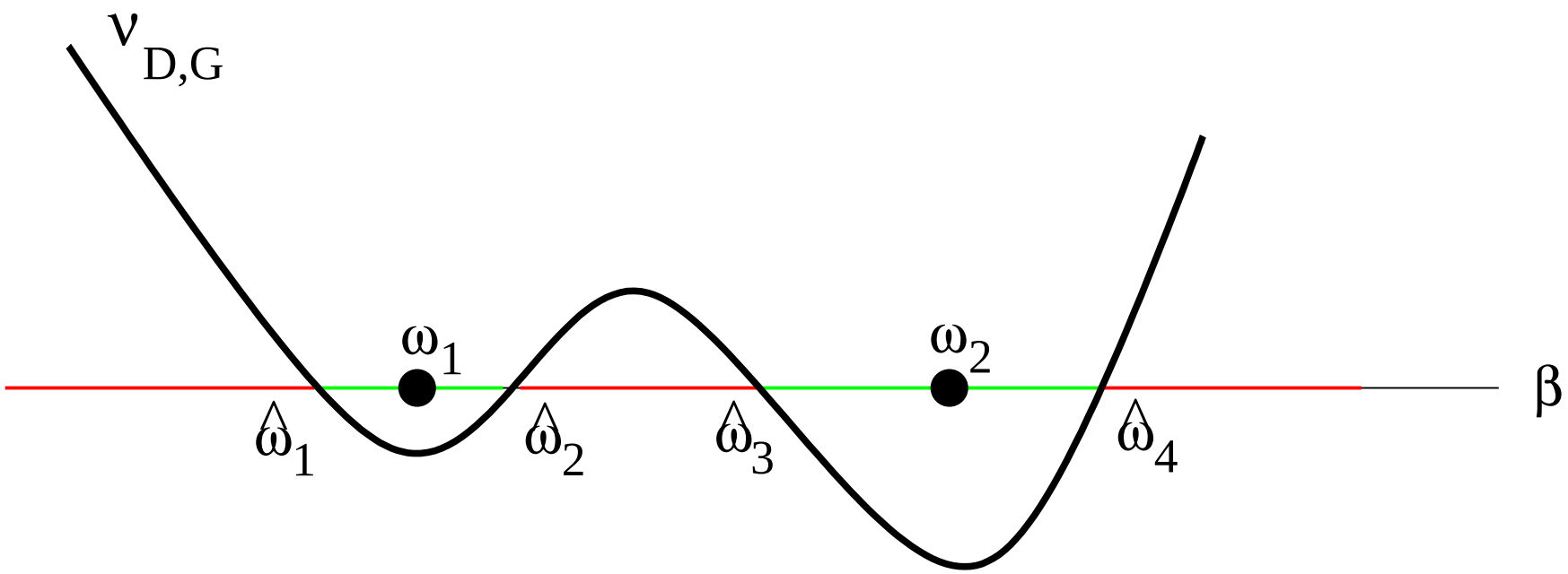
then, for all $\beta \geq \hat{\beta}$ the inequality (*) holds.

$\square$

- Note that the above result is computationaly attractive since all solutions of (**) can be easily found by computing generalized eigenvalues.
- A tolerance parameter ϵ_{tol} can be defined to decide when ω_1 and ω_2 are not close enough, *i.e* :

Validity of a frequency segment

A preliminary illustration



Validity of a frequency segment

A related technical result

Given ω_1, ω_2, β, D and G as introduced above, define $\omega_0 = (w_1 + w_2)/2$ then there exists an augmented matrix $\mathcal{H}(\omega_0)$ whose **real** eigenvalues $\eta_1, \ldots, \eta_q$ have the following property :

$\forall k = 1, \ldots, q \;\; \exists i$ such that :

$$\sigma_i \left(F^{-1/4} \left(\frac{DM(j\hat{\omega}_k)D^{-1}}{\beta} - jG \right) F^{-1/4} \right) = 1$$

with :

$$\hat{\omega}_k = \omega_0 + \frac{1}{\eta_k}$$

$\square$

Validity of a frequency segment

Corollary

Consider the frequency segment $I_k = [\hat{\omega}_k \;\; \hat{\omega}_{k+1}]$, and define a frequency point $\check{\omega}_k$ strictly inside. If

$$\overline{\sigma}\left(F^{-1/4}\left(\frac{DM(j\check{\omega}_k)D^{-1}}{\beta} - jG\right)F^{-1/4}\right) \leq 1$$

then, it holds true $\forall \omega \in I_k$.

$\square$

Proposed algorithm

Define an initial frequency gridding $\{\omega_1, \omega_2, \ldots, \omega_N\}$ *and generate the associated set of intervals :*

$$\mathcal{I} = \{I_k = [\omega_k\,,\ \omega_{k+1}]\,, k = 1, \ldots, N-1\}$$

then repeat the following steps while $\mathcal{I}$ *is not empty :*

1. Consider an interval I_k of $\mathcal{I}$ and denote $\overline{\omega}_k = (\omega_k + \omega_{k+1})/2$,
2. By the two-step procedure, compute β_k, D_k and G_k which are simultaneously valid for ω_k and ω_{k+1},
3. If $\beta_k - \beta_{0_k} > \epsilon_{tol}$, add the central point $\overline{\omega}_k$ to the gridding, update $\mathcal{I}$ and go back to first step. Otherwise, update β_{max} :

$$\beta_{max} \leftarrow max(\beta_{max}, \beta_k)$$

4. Compute the set $\mathcal{J}$ of frequency intervals inside which β_{max}, D_k and G_k are valid,
5. Update $\mathcal{I}$ by removing all intervals of $\mathcal{J}$:

$$\mathcal{I} \leftarrow \mathcal{I} - \mathcal{J}$$

Proposed algorithm

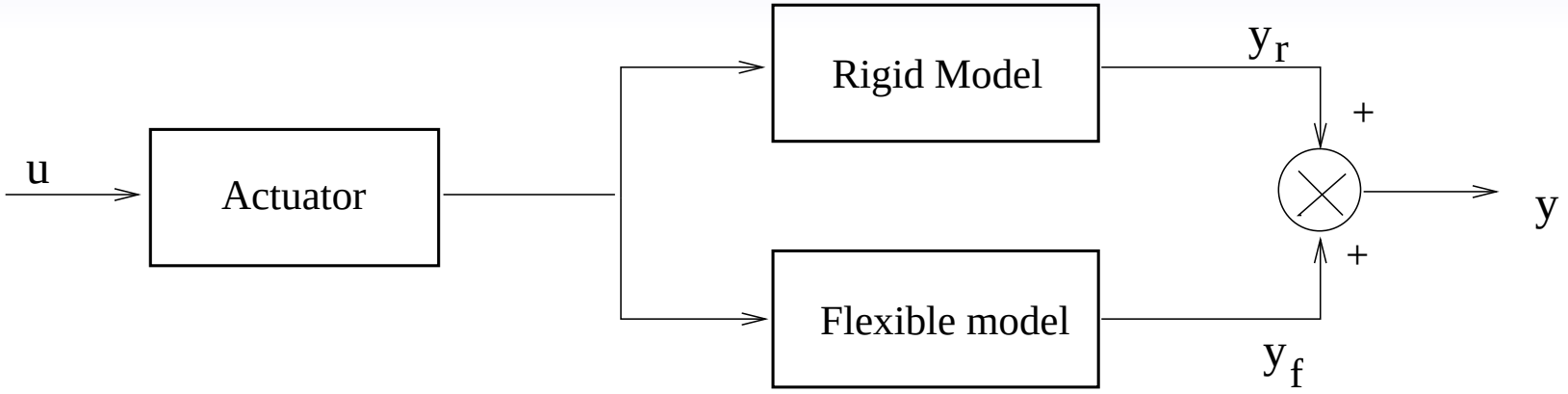
Possible improvements

- Improving accuracy by switching to the LMI technique on most critical frequency segments,
- Reducing computational-time by first computing a μ lower-bound (μ_L) and then initializing β_{max} :

$$\beta_{max} \leftarrow \mu_L$$

Application to the flexible aircraft

Model description



Rigid part :

Linearized lateral model with uncertainties in the 14 stability derivatives :

$$
\begin{aligned}
\dot{\beta} &= Y_\beta \beta + (Y_p + sin\alpha_0)p + (Y_r - cos\alpha_0)r + \frac{g}{V}\phi + Y_{\delta p}\delta p + Y_{\delta r}\delta r \\
\dot{p} &= L_\beta \beta + L_p p + L_r r + L_{\delta p}\delta p + L_{\delta r}\delta r \\
\dot{r} &= N_\beta \beta + N_p p + N_r r + N_{\delta r}\delta r \\
\dot{\phi} &= p + tan\theta_0 \; r
\end{aligned}
$$

Application to the flexible aircraft

Model description

Flexible part

The 12th order flexible model contains 6 poorly damped bending modes, which caracteristics are summarized in the following table :

	damping ratio	natural frequency (rad/s)
1	$1.56\,10^{-2}$	14.3
2	$2.16\,10^{-2}$	13.5
3	$2.42\,10^{-2}$	12.5
4	$3.29\,10^{-2}$	7.35
5	$5.07\,10^{-2}$	14.1
6	$5.09\,10^{-2}$	8.62

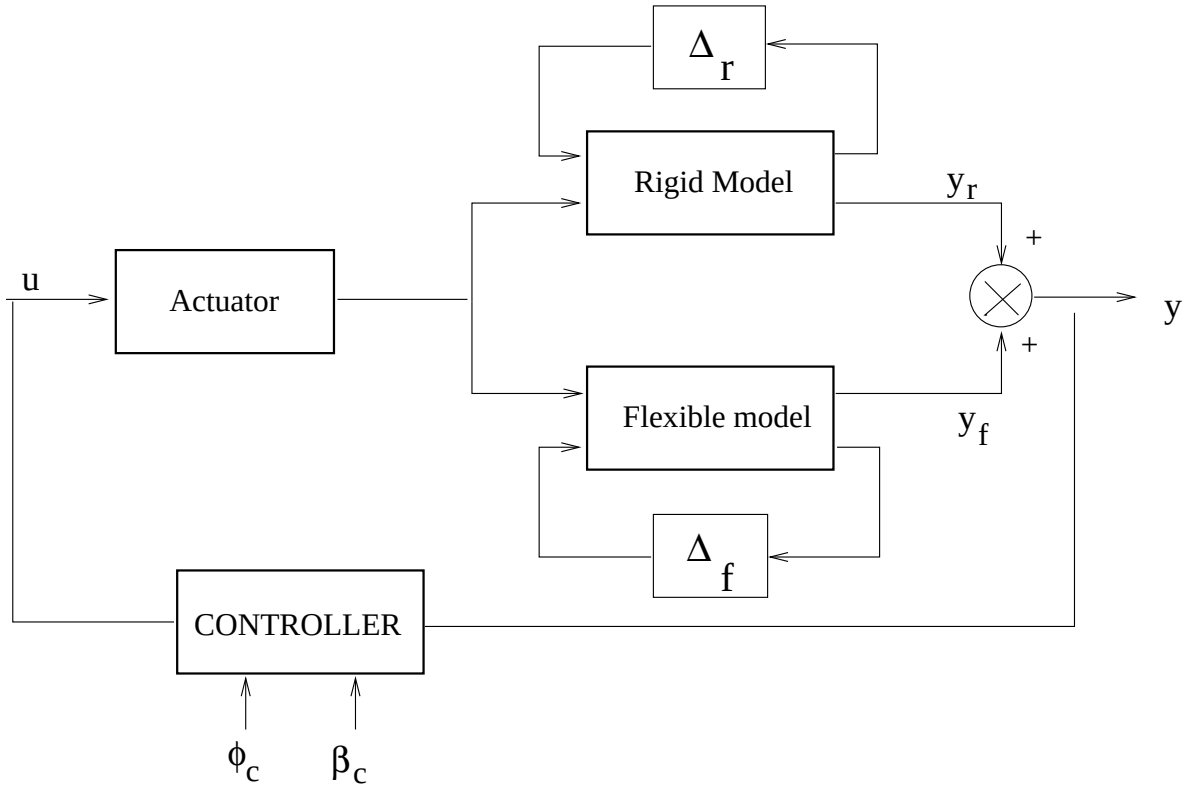
Application to the flexible aircraft

Controller design

We used an observer-based state-feedback control law. The idea is simply to place the rigid closed-loop poles to achieve performances and decoupling objectives. Then 4 flexibles modes are actively controlled (they are shifted into the LHP in order to improve their damping). Moreover, roll-off filters are added to the design model in order to increase the robustness properties of the control law versus unmoddelled dynamics.

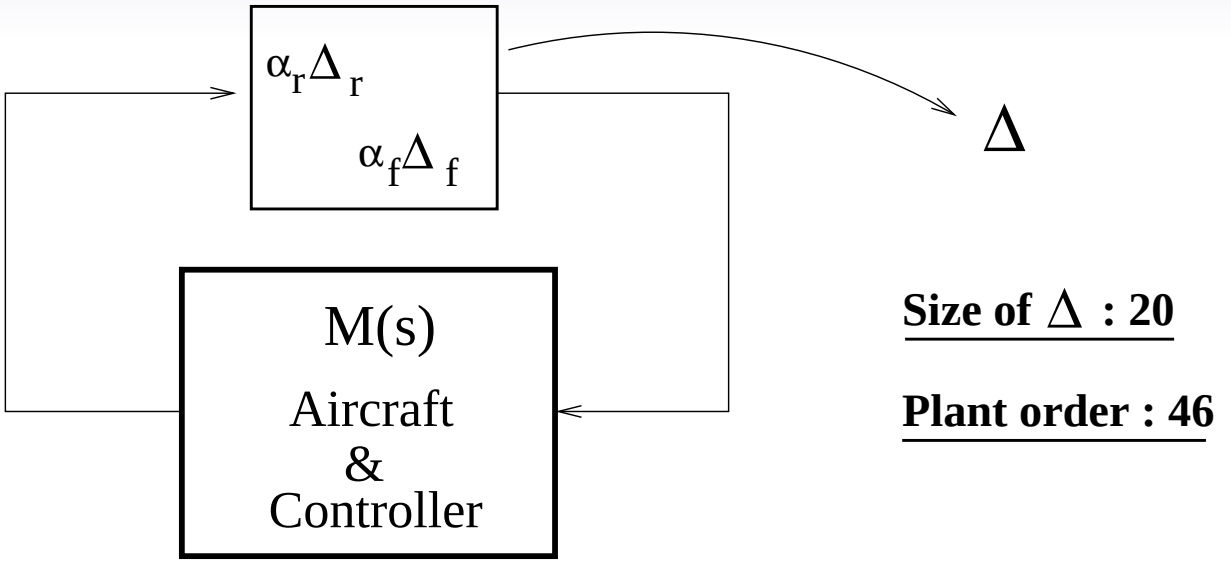
Application to the flexible aircraft

LFT for the closed-loop



Application to the flexible aircraft

Global and scaled LFT



$$\Delta = \mathbf{diag}\left(\delta_{r_1}, \ldots \delta_{r_{14}}, \delta_{f_1}, \ldots, \delta_{f_6}\right)$$

Scaling parameters α_r and α_f are tuned in a way that if the robustness margin of the system is less than one, then the maximum uncertainty levels are :

- 10% on the stability derivatives

Application to the flexible aircraft

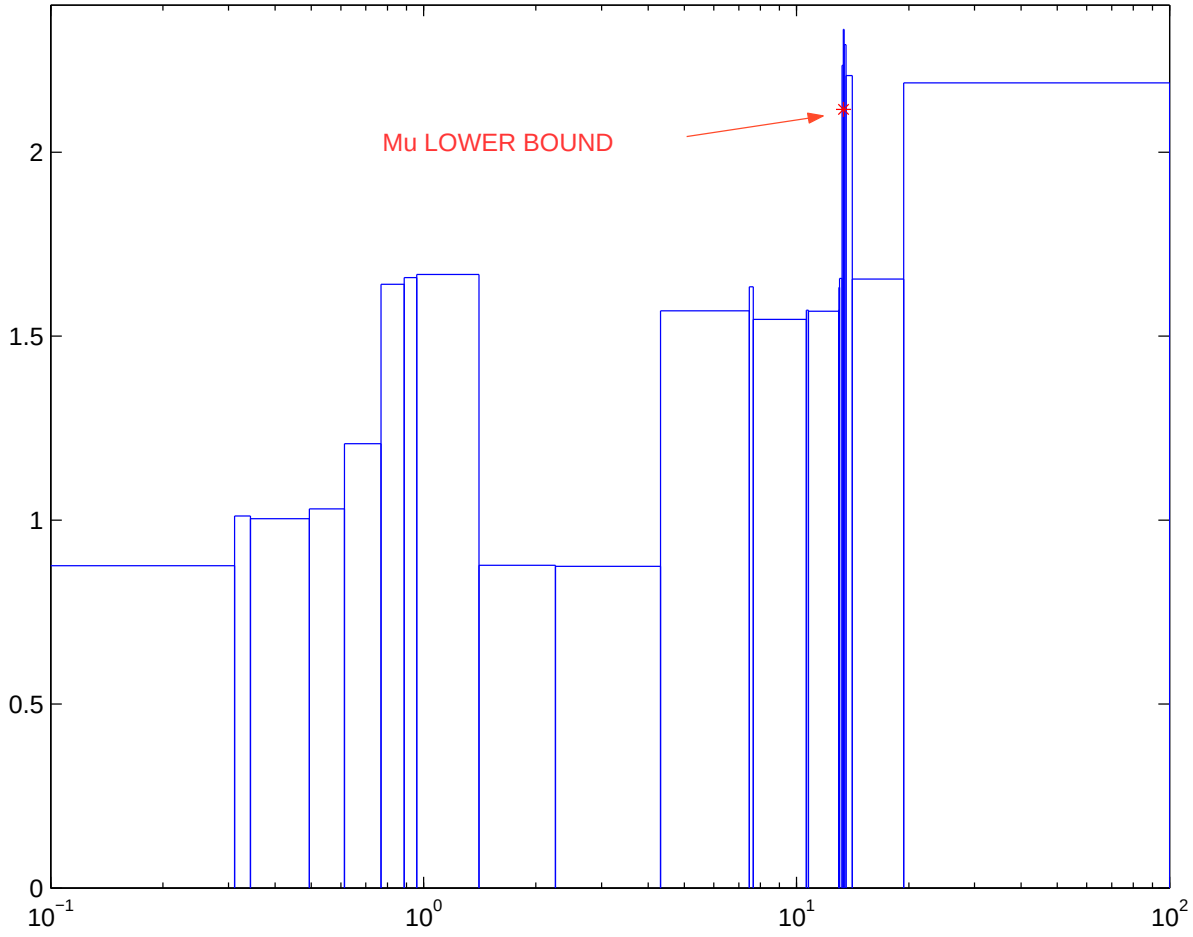
Application of the routine : **mub = mu_max_3(sys,blk) ;**

Initial steps : $13\,s$ LMI steps : $5\,s$

Iter.	Remaining Int.	$\overline{\mu}$ so far	reliability
1	20	0	0%
2	21	0	0%
3	22	0.88	0.14%
...	...	...	...
25	15	1.67	12.6%
...	...	...	...
37	5	2.41	100%
1	1	2.34	0%
2	2	2.34	50%
3	1	2.34	100%

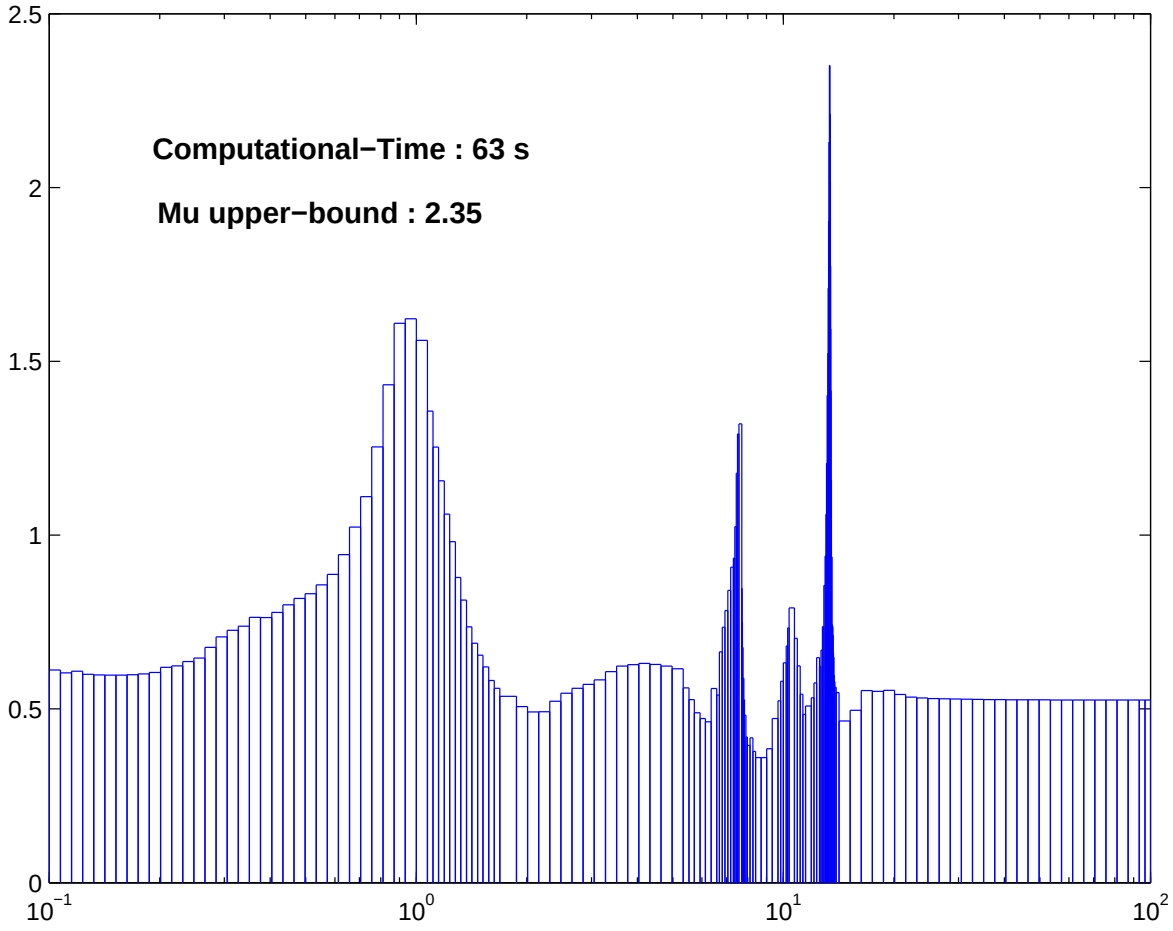
Application to the flexible aircraft

A graphical illustration of the results :



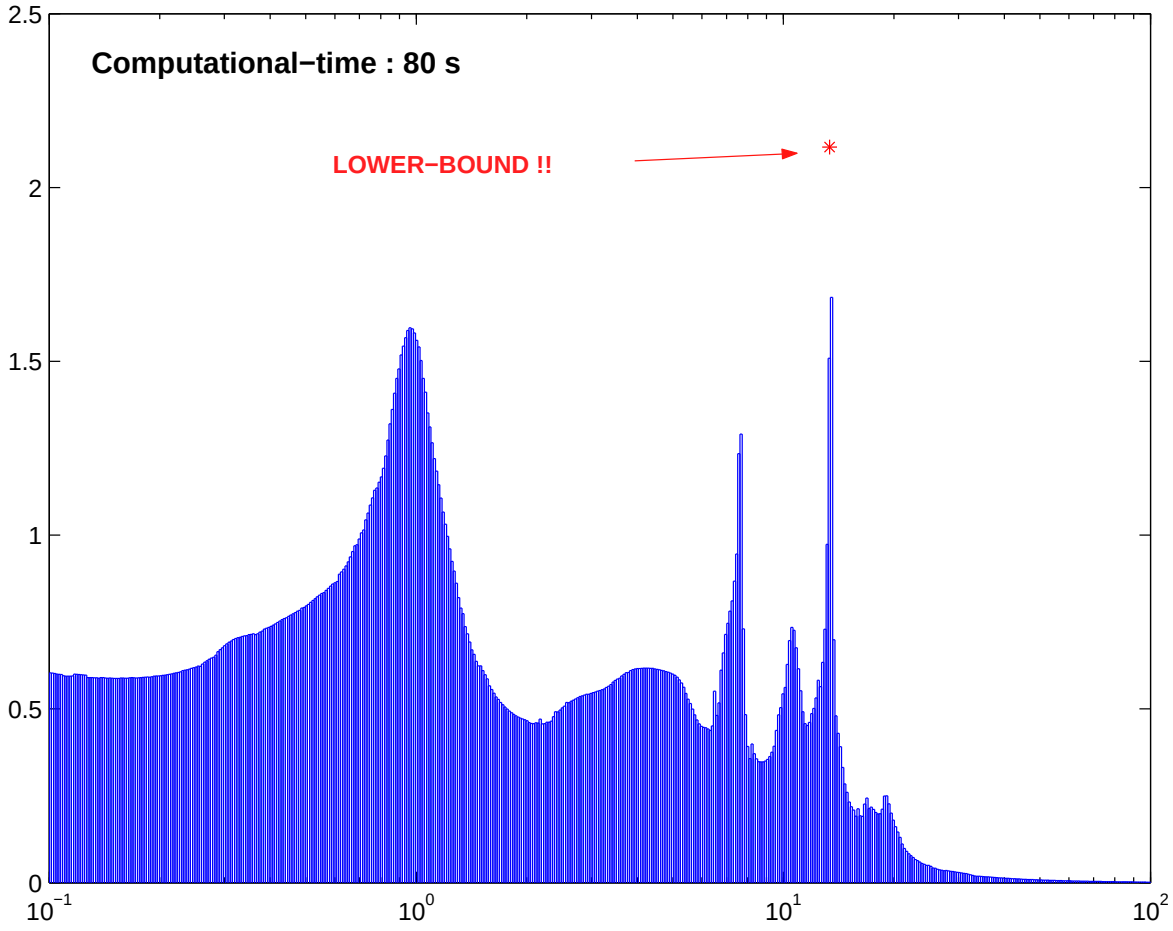
Application to the flexible aircraft

A more precise computation ...



Application to the flexible aircraft

A classical gridding-based approach with 500 points



Application to the flexible aircraft

Conclusion on the methods

- Using our method, an accurate and **reliable** μ upper-bound was found in less than $20\,s$ despite the complexity of the problem,
- A classical gridding-based approached failed : The peak was missed despite the fine gridding we used. Furthermore, computational-time is higher.

Conclusion about the FCS

We have proved that the FCS can tolerate an uncertainty of :

- $4.25\,\%$ in the stability derivatives,
- $8.5\,\%$ in the natural frequencies of the bending modes.

which is rather lower and suggests that the controller should be further tuned.

Conclusion

- A new, fast and reliable method has been presented to evaluate the robustness margin of a system in presence of parametric uncertainties and/or neglected dynamics.
- It works on high-order badly-dampled systems with numerous, possibly largely repeated uncertainties.
- It is available as part of the **Skew-μ Toolbox** (SMT) :

http ://www.onera.fr/staff/jean-marc-biannic